Secondary School Examination-2020

Marking Scheme - Mathematics 30/5/1, 30/5/2, 30/5/3

General instructions

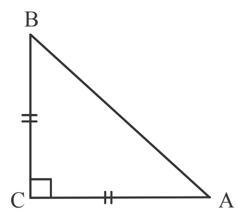
1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best effortsin this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks __________(example 0-100 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/5/1
EXPECTED ANSWER/VALUE POINTS
SECTION – A

Question numbers 1 to 20 are of 1 mark each.

Question numbers **1** to **10** are multiple choice questions.

You have to select the correct choice :

Q.No.		Marks
1.	On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is **(a)** $3x^2 + x - 12$ **(b)** $x^3 - 4x + 3$ **(c)** $x^2 + 3x - 4$ **(d)** $x^3 - 4x - 3$	
	Ans: (b) $x^3 - 4x + 3$	1
2.	In Figure 1, ABC is an isosceles triangle, right-angled at C. Therefore **(a)** $AB^2 = 2AC^2$ **(b)** $BC^2 = 2AB^2$ **(c)** $AC^2 = 2AB^2$ **(d)** $AB^2 = 4AC^2$  Figure 1	
	Ans: (a) $AB^2 = 2AC^2$	1
3.	The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is **(a)** (7, 0) **(b)** (5, 0) **(c)** (0, 0) **(d)** (3, 0)	
	Ans: (d) (3, 0)	1
	OR	
	The centre of a circle whose end points of a diameter are (–6,3) and (6, 4) is **(a)** (8, –1) **(b)** (4, 7) **(c)** $\left(0, \frac{7}{2}\right)$ **(d)** $\left(4, \frac{7}{2}\right)$	
	Ans: (c) $\left(0, \frac{7}{2}\right)$	1
4.	The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is **(a)** 4 **(b)** ±4 **(c)** –4 **(d)** 0	
	Ans: (b) ± 4	1
5.	Which of the following is *not* an A.P.? **(a)** –1.2, 0.8, 2.8, … **(b)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$ **(c)** $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$ **(d)** $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$	
	Ans: (c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$	1

6. The pair of linear equations

$\frac{3x}{2}+\frac{5y}{3}=7$ and $9x + 10y = 14$ is

(a) consistent

(b) inconsistent

(c) consistent with one solution

(d) consistent with many solutions

Ans: (b) inconsistent 1

7. In Figure 2, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

(a) 50°

(b) 40°

(c) 60°

(d) 80°

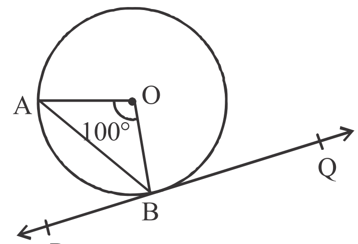


Figure 2

Ans: (a) 50° 1

8. The radius of a sphere (in cm) whose volume is 12π cm^3, is

(a) 3

(b) $3\sqrt{3}$

(c) $3^{2/3}$

(d) $3^{1/3}$

Ans: (c) $3^{2/3}$ 1

9. The distance between the points (m, –n) and (–m, n) is

(a) $\sqrt{m^2+n^2}$

(b) m + n

(c) $2\sqrt{m^2+n^2}$

(d) $\sqrt{2m^2+2n^2}$

Ans: (c) $2\sqrt{m^2+n^2}$ 1

10. In Figure 3, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is

(a) 3 cm

(b) 4 cm

(c) 2 cm

(d) $2\sqrt{2}$ cm

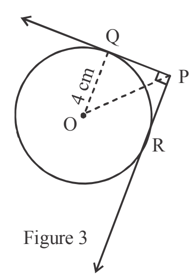


Figure 3

Ans: (b) 4 cm 1

Fill in the blanks in question numbers 11 to 15.

11. The probability of an event that is sure to happen, is __________.

Ans: 1 1

12. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is __________.

Ans: $\tan^2 A$ 1

13. AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is __________.

Ans: 5 units 1

14. In the formula $\overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$, u_i = ________.

Ans: $\frac{x_i - a}{h}$ — **1**

15. All concentric circles are ________ to each other.

Ans: similar — **1**

Answer the following question numbers 16 to 20.

16. Find the sum of the first 100 natural numbers.

Ans: $\frac{100}{2}[2+99] = 5050$ — **1/2+1/2**

17. In Figure 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of tower.

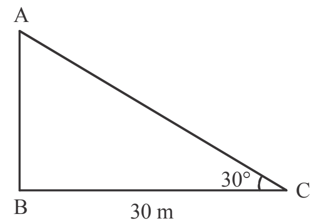


Figure 4

Ans: $\frac{AB}{30} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m — **1/2+1/2**

18. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

Ans: $\frac{182 \times 13}{26} = 91$ — **1/2+1/2**

19. Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively.

Ans: $x^2 + 3x + 2$ — **1**

OR

Can $(x^2 – 1)$ be a remainder while dividing $x^4 – 3x^2 + 5x – 9$ by $(x^2 + 3)$? Justify your answer with reasons.

Ans: No, degree of remainder < degree of divisor — **1**

20. Evaluate : $\frac{2 \tan 45° \times \cos 60°}{\sin 30°}$

Ans: $\frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = 2$ — **1/2+1/2**

SECTION – B

Question numbers 21 to 26 carry 2 marks each.

21. In the given Figure 5, DE || AC and DF || AE.

Prove that $\frac{BF}{EF} = \frac{BE}{EC}$.

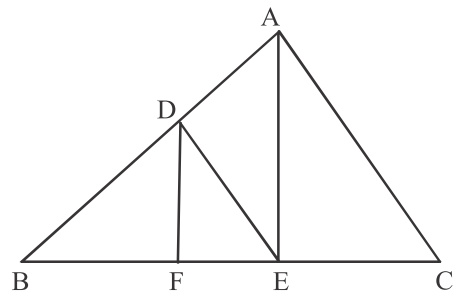


Figure 5

	Ans: In ΔABE, $DF \parallel AE$, $\therefore \frac{BD}{AD} = \frac{BF}{FE}$ … (i)	1
	In ΔABC, $DE \parallel AC$, $\therefore \frac{BD}{AD} = \frac{BE}{EC}$ … (ii)	1/2
	From (i) and (ii) $\frac{BF}{FE} = \frac{BE}{EC}$	1/2
22.	Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.	
	Ans: Let us assume that $5+2\sqrt{7}$ is not an irrational number.	
	$\therefore$ $5+2\sqrt{7}$ is a rational number p i.e. $5+2\sqrt{7} = p$	1
	$\Rightarrow \sqrt{7} = \frac{p-5}{2}$	1/2
	Which is a contradiction as RHS is a rational but LHS is irrational.	
	Hence $5+2\sqrt{7}$ can not be rational, so irrational.	1/2
	OR	
	Check whether 12^n can end with the digit 0 for any natural number n.	
	Ans: Prime factors of 12 are $2 \times 2 \times 3$	1
	Since 5 is not a factor, so 12^n can not end with 0.	1
23.	If A, B and C are interior angles of a ΔABC, then show that $\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$.	
	Ans: $A + B + C = 180°$, $\therefore \frac{B+C}{2} = 90° - \frac{A}{2}$	1
	$\therefore \cot\left(\frac{B+C}{2}\right) = \cot\left(90° - \frac{A}{2}\right) = \tan\frac{A}{2}$	1

24.	In Figure 6, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.	

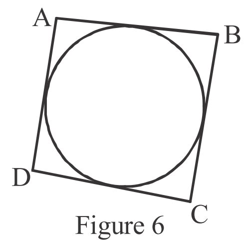
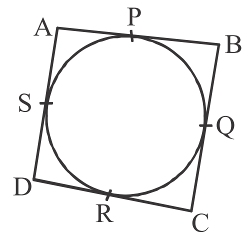


Figure 6

Ans: Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively. **1/2**



$\therefore$ AP = AS
BP = BQ
DR = DS
CR = CQ **1**

adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ) **1/2**

$\therefore$ AB + CD = BC + AD

OR

In Figure 7, find the perimeter of ΔABC, if AP = 12 cm.

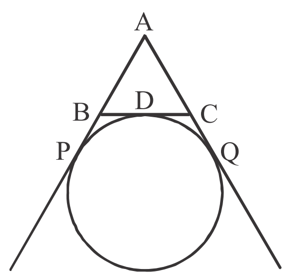


Figure 7

Ans: AP = AB + BP = AB + BD
AQ = AC + CQ = AC + CD **1**

$\Rightarrow$ AP + AQ = AB + AC + (BD + CD) = AB + AC + BC

But AP = AQ $\therefore$ 2 AP = Perimeter of ABC **1/2**

$\therefore$ Perimeter = 2(12) = 24 cm **1/2**

25. Find the mode of the following distribution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students:	4	6	7	12	5	6

Ans: Modal Group : 30 – 40 **1/2**

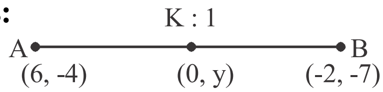
$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 30 + \frac{5}{12} \times 10$$ **1**

$= 34.17$ **1/2**

26.	2 cubes, each of volume 125 cm^3, are joined end to end. Find the surface area of the resulting cuboid.	
	Ans: Side of cube = $(125)^{1/3} = 5$ cm	1/2
	$\therefore$ Dimensions of cuboid : 10, 5, 5	1/2
	S.A = 2(50 + 25 + 50) = 250 cm^2	1
	SECTION – C	
	Question numbers 27 to 34 carry 3 marks each.	
27.	A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.	
	Ans: Let the fraction be $\frac{x}{y}$	1/2
	$\therefore \frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$	1/2+1/2
	$\Rightarrow 3x - y = 3 , 4x - y = 8$	1/2
	Solving to get x = 5, y = 12 $\therefore$ Fraction is $\frac{5}{12}$	1
	OR	
	The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.	
	Ans: Let the present age of son be x years	
	$\therefore$ Father's present age = (3x + 3) years.	1
	3 years hence, Son's age = (x + 3) years and father's age = (3x + 6) years	1/2
	$\therefore$ 3x + 6 = 2(x + 3) + 10	1
	$\Rightarrow$ x = 10 $\therefore$ Son's age = 10 years, Father's age = 33 years	1/2
28.	Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.	
	Ans: Any positive integer 'n' can be of the form 3m, 3m + 1, 3m + 2 (for some integer m)	1
	$\therefore n^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3q$, or $n^2 = (3m + 1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1 = 3q + 1$, or $n^2 = (3m + 2)^2 = 9m^2 + 12m + 3 + 1$ $= 3(3m^2 + 4m + 1) + 1 = 3q + 1$	$1\frac{1}{2}$
	Hence square of any positive integer is either of the form 3q or 3q + 1 for some integer q.	1/2

29. Find the ratio in which the y-axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

Ans:



Let the point P(0, y) on y-axis divides the line segment AB in K : 1 **1**

$$\therefore \quad 0 = \frac{-2K+6}{K+1} \Rightarrow K = 3 \quad \therefore \text{ Ratio is } 3:1$$ **1**

Also, $y = \frac{3(-7)+1(-4)}{3+1} = \frac{-25}{4}$ $\therefore$ Point of intersection is $\left(0, \frac{-25}{4}\right)$ **1**

OR

Show that the points $(7, 10)$, $(-2, 5)$ and $(3, -4)$ are vertices of an isosceles right triangle.

Ans: Let the points be A(7, 10), B(–2, 5) and C(3, –4)

$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$ **1**

$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$ **1/2**

$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$ **1/2**

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle. **1**

30. Prove that: $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Ans: $LHS = \sqrt{\frac{1+\sin A}{1-\sin A} \cdot \frac{1+\sin A}{1+\sin A}}$ **1**

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$ **$1+\frac{1}{2}$**

$= \sec A + \tan A$ **1/2**

31. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the n^{th} term $(a_n) = 50$. Find n and sum of first n terms (S_n) of the A.P.

Ans: $50 = 5 + (n-1)3 \Rightarrow n = 16$ **$1+\frac{1}{2}$**

$S_{16} = \frac{16}{2}[10 + 15 \times 3] = 440$ **$1+\frac{1}{2}$**

32. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Ans: Constructing ΔABC with given dimensions **1**

Constructing the similar triangle. **2**

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Ans: Drawing a circle of radius 3.5 cm and centre O, and taking a point P such that OP = 7 cm **1**

Constructing two tangents. **2**

33. Read the following passage and answer the questions given at the end:

Diwali Fair

A game in booth at Diwali fair involves using of spinner first. Then, if the spinner stops at an even number, the player is allowed to pick a marble from bag. The spinner and the marbles in the bag are represented in Figure-8

Prizes are given, when a black marble is picked. Shweta plays the game once.

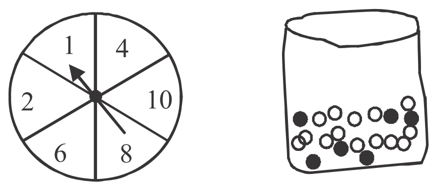


Figure 8

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Ans: (i) P(she will be allowed to pick a marble) = $\frac{5}{6}$ **$1\frac{1}{2}$**

(ii) P(getting a prize) = $\frac{6}{20}$ or $\frac{3}{10}$ **$1\frac{1}{2}$**

Both answers $\frac{6}{20}$ or $\frac{0}{20}$ for part (ii) in Q33 are to be treated correct as the bag contains marbles only.

34. In Figure-9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of the circle is $6\sqrt{2}$ cm, find the area of shaded region.

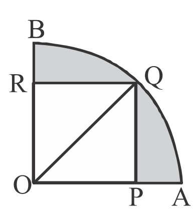


Figure 9

	Ans: Let side of square be 'a' cm $\therefore\ a^2 + a^2 = \left(6\sqrt{2}\right)^2 \Rightarrow a = 6$ cm	1
	$\therefore$ Area of shaded region $= \pi r^2 \frac{90}{360} - a^2 = \frac{22}{7} \times \left(6\sqrt{2}\right)^2 \cdot \frac{1}{4} - 36$	$1+\frac{1}{2}$
	$= \frac{396-252}{7} = \frac{144}{7}\text{cm}^2$ or $20.57\ \text{cm}^2$	1/2
	SECTION – D	
	Question numbers 35 to 40 carry 4 marks each.	
35.	Obtain other zeroes of the polynomial $P(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ If two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.	
	Ans: Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x), so $\left(x-\sqrt{5}\right)$ and $\left(x+\sqrt{5}\right)$ are factors of p(x). Thus $(x^2 - 5)$ is a factor of p(x).	1
	$\left(2x^4 - x^3 - 11x^2 + 5x + 5\right) \div \left(x^2 - 5\right) = 2x^2 - x - 1$	$1\frac{1}{2}$
	$2x^2 - x - 1 = (2x + 1)(x - 1)$	1
	$\therefore$ Other zeroes of p(x) are $1, -\frac{1}{2}$	1/2
	OR	
	What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?	
	Ans:	
	$$\begin{array}{r\|l} & 2x+5 \\ x^2-4x+8 & 2x^3-3x^2+6x+7 \\ - & 2x^3-8x^2+16x \\ & (-)\ \ (+)\ \ (-) \\ \hline & 5x^2-10x+7 \\ - & 5x^2-20x+40 \\ & (-)\ \ (+)\ \ (-) \\ \hline & 10x-33 \end{array}$$	3
	$\therefore$ We have to add $(33 - 10x)$	1
36.	Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.	
	Ans: For correct Given, To Prove, Constructions and figure	$\frac{1}{2}\times 4 = 2$
	For correct proof	2

37. Sum of the areas of 2 squares is 544 m^2. If the difference of their perimeter is 32 m, find the sides of two squares.

Ans: Let 'a' and 'b' be the sides of two squares, with a > b.

then $a^2 + b^2 = 544$ and $4a - 4b = 32$ — $1\frac{1}{2}$

or $a - b = 8 \;\therefore\; a = b + 8$

$\therefore\; (b+8)^2 + b^2 = 544 \;\Rightarrow\; 2b^2 + 16b - 480 = 0$ — 1

$\therefore\; b^2 + 8b - 240 = 0 \;\Rightarrow\; (b+20)(b-12) = 0 \;\Rightarrow\; b = 12$ — 1

$b = 12$ m $\Rightarrow\; a = 12 + 8 = 20$ m — 1/2

OR

A motorboat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans: Let speed of the stream be x km/h

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ — 2

$\Rightarrow\; 24(2x) = 324 - x^2$ or $x^2 + 48x - 324 = 0$ — 1

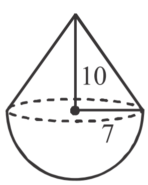
$\Rightarrow\; (x+54)(x-6) = 0 \;\Rightarrow\; x = 6$ — 1

$\therefore$ Speed of the stream = 6 km/h

38. A solid toy in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of its base is 7 cm. Determine the volume of the toy. Also find the area of the colored sheet required to cover the toy.

(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

Ans: Volume of toy $= \frac{2}{3}\pi(7)^3 + \frac{1}{3}\pi(7)^2 \times 10\,\text{cm}^3$ — 1

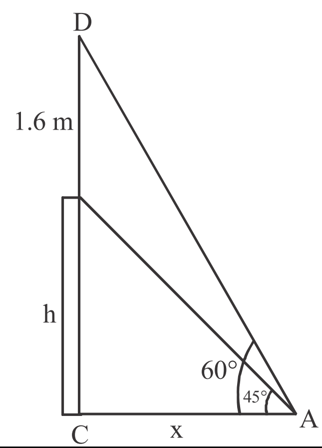


$= \frac{1}{3} \times \frac{22}{7} \times 49\,(14 + 10) = 1232\,\text{cm}^3$ — 1

Area of Sheet = Surface area $= 2\pi(7)^2 + \pi(7)\sqrt{10^2 + 7^2}$ — 1

$= 308 + 22 \times 12.2 = 576.4\text{cm}^2$ — 1

39. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statute is 60° and from the same point the angle of elevation of the top of pedestal id 45°. Find the height of the pedestal. (Use $\sqrt{3} = 1.73$)



Ans: For correct figure. — 1

Let h m be the height of pedestal

Then from figure, $\frac{h}{x} = \tan 45° = 1$

and $\frac{h+1.6}{x} = \tan 60° = \sqrt{3}$ — 1 + 1

$\Rightarrow\; \frac{h+1.6}{h} = \sqrt{3} \;\Rightarrow\; \left(\sqrt{3} - 1\right)h = 1.6$ — 1/2

$\Rightarrow h = \frac{160}{73} = 2.19$ m (approx) **1/2**

40. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (In years):	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

Ans: The points to be plotted for less than ogive are

(10, 5), (20, 20), (30, 40), (40, 65), (50 , 80), (60, 91), (70, 100) **2**

Drawing the ogive **$1\frac{1}{2}$**

Getting median = 34 (approx) **1/2**

OR

The distribution given below shows that the number of wickets taken by bowler in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20-60	60-100	100-140	140-180	180-220	230-260
Number of bowlers :	7	5	16	12	2	3

Ans:

No. of wickets :	20-60	60-100	100-140	140-180	180-220	220-260	Sum	
(f_i) No. of bowlers :	7	5	16	12	2	3	45	
x_i	40	80	120	160	200	240		
u_i	−2	−1	0	1	2	3		**1/2**
$f_i x_i$	−14	−5	0	12	4	9	6	**1/2**
cf	7	12	28	40	42	45		**1/2**

Mean = $a + \frac{\sum f_i u_i}{\sum f_i} \times h = 120 + \frac{6 \times 40}{45} = 125.33$ **$1\frac{1}{2}$**

Median = $l + \frac{\frac{N}{2} - c}{f} \times h = 100 + \frac{22.5 - 12}{16} \times 40 = 126.25$ **1**

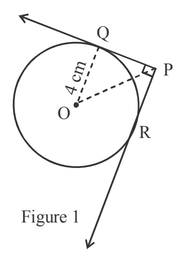
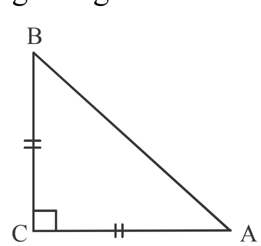
QUESTION PAPER CODE 30/5/2

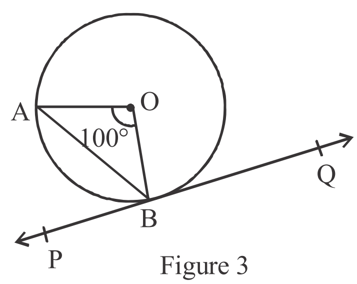
EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is	
	(a) 4 **(b)** ± 4 **(c)** -4 **(d)** 0	
	Ans: (b) ± 4	1
2.	Which of the following is *not* an A.P.?	
	(a) $-1.2, 0.8, 2.8, \ldots$ **(b)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$	
	(c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$ **(d)** $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$	
	Ans: (c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$	1
3.	In Figure 1, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is	
	(a) 3 cm	
	(b) 4 cm	
	(c) 2 cm	
	(d) $2\sqrt{2}$ cm	
	 Figure 1	
	Ans: (b) 4 cm	1
4.	The distance between the points (m, –n) and (–m, n) is	
	(a) $\sqrt{m^2 + n^2}$ **(b)** $m + n$	
	(c) $2\sqrt{m^2 + n^2}$ **(d)** $\sqrt{2m^2 + 2n^2}$	
	Ans: (c) $2\sqrt{m^2 + n^2}$	1
5.	The degree of the polynomial having zeroes –3 and 4 only is	
	(a) 2 **(b)** 1	
	(c) More than 3 **(d)** 3	
	Ans: All the three options (a), (c) and (d) are acceptable	
	1 mark for any of the option (a), (c) or (d)	1
6.	In Figure 2, ABC is an isosceles triangle, right-angled at C. Therefore	
	(a) $AB^2 = 2AC^2$	
	(b) $BC^2 = 2AB^2$	
	(c) $AC^2 = 2AB^2$	
	(d) $AB^2 = 4AC^2$	
	 Figure 2	
	Ans: (a) $AB^2 = 2AC^2$	1

7.	The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is **(a)** (7, 0) **(b)** (5, 0) **(c)** (0, 0) **(d)** (3, 0) **Ans:** (d) (3, 0)	**1**
	OR The centre of a circle whose end points of a diameter are (–6,3) and (6, 4) is **(a)** (8, –1) **(b)** (4, 7) **(c)** $\left(0, \frac{7}{2}\right)$ **(d)** $\left(4, \frac{7}{2}\right)$ **Ans:** (c) $\left(0, \frac{7}{2}\right)$	**1**
8.	The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is **(a)** consistent **(b)** inconsistent **(c)** consistent with one solution **(d)** consistent with many solutions **Ans:** (b) inconsistent	**1**
9.	In Figure 3, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to **(a)** 50° **(b)** 40° **(c)** 60° **(d)** 80° Figure 3 **Ans:** (a) 50°	**1**
10.	The radius of a sphere (in cm) whose volume is 12π cm^3, is **(a)** 3 **(b)** $3\sqrt{3}$ **(c)** $3^{2/3}$ **(d)** $3^{1/3}$ **Ans:** (c) $3^{2/3}$	**1**
	Fill in the blanks in question numbers 11 to 15.	
11.	AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is __________. **Ans:** 5 units	**1**
12.	In the formula $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$, u_i = __________. **Ans:** $\frac{x_i - a}{h}$	**1**
13.	All concentric circles are __________ to each other. **Ans:** similar	**1**
14.	The probability of an event that is sure to happen, is __________. **Ans:** 1	**1**

15. Simplest form of $(1 - \cos^2 A)(1 + \cot^2 A)$ is _________.

Ans: 1 — **1**

Answer the following question numbers 16 to 20.

16. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

Ans: $\frac{182 \times 13}{26} = 91$ — **1/2+1/2**

17. Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively.

Ans: $x^2 + 3x + 2$ — **1**

OR

Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$? Justify your answer with reasons.

Ans: No, degree of remainder < degree of divisor — **1**

18. Find the sum of the first 100 natural numbers.

Ans: $\frac{100}{2}[2 + 99] = 5050$ — **1/2+1/2**

19. Evaluate:

$2 \sec 30° \times \tan 60°$

Ans: $2 \times \frac{2}{\sqrt{3}} \times \sqrt{3}$ — **1/2**

$= 4$ — **1/2**

20. In Figure 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

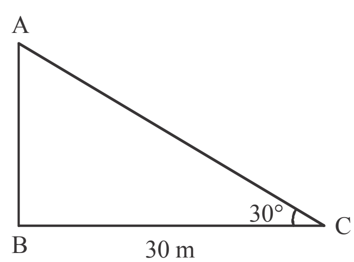


Figure 4

Ans: $\frac{AB}{30} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m — **1/2+1/2**

SECTION – B

Question numbers 21 to 26 carry 2 marks each.

21. Find the mode of the following distribution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students:	4	6	7	12	5	6

Ans: Modal class : 30 – 40 — **1/2**

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 30 + \frac{5}{12} \times 10$$ — **1**

$= 34.17$ — **1/2**

22. In Figure 5, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

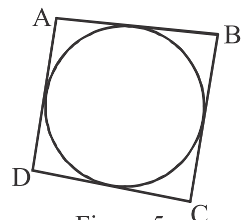
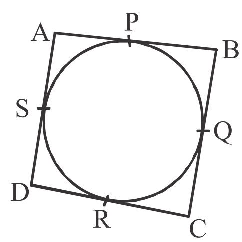


Figure 5

Ans: Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively. **1/2**



$\therefore$ AP = AS
BP = BQ
DR = DS
CR = CQ **1**

adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ) **1/2**

$\therefore$ AB + CD = BC + AD

OR

In Figure 6, find the perimeter of ΔABC, if AP = 12 cm.

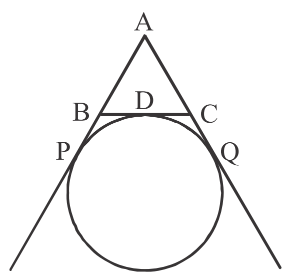


Figure 6

Ans: AP = AB + BP = AB + BD
AQ = AC + CQ = AC + CD **1**

$\Rightarrow$ AP + AQ = AB + AC + (BD + CD) = AB + AC + BC

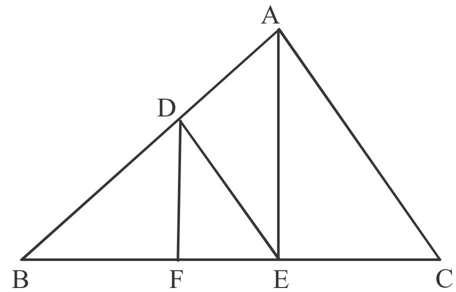
But AP = AQ $\therefore$ 2 AP = Perimeter of ABC **1/2**

$\therefore$ Perimeter = 2(12) = 24 cm **1/2**

23. How many cubes of side 2 cm can be made from a solid cube of side 10 cm?

Ans: No. of cubes = $\frac{10\times10\times10}{2\times2\times2}$ **1**

= 125 **1**

24.	In the given Figure 7, DE ∥ AC and DF ∥ AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.  Figure 7	
	Ans: In ΔABE, DF ∥ AE, $\therefore \frac{BD}{AD} = \frac{BF}{EF}$ … (i)	1
	In ΔABC, DE ∥ AC, $\therefore \frac{BD}{AD} = \frac{BE}{EC}$ … (ii)	1/2
	From (i) and (ii) $\frac{BF}{FE} = \frac{BE}{EC}$	1/2
25.	Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.	
	Ans: Let us assume that $5+2\sqrt{7}$ is not an irrational number.	
	$\therefore$ $5+2\sqrt{7}$ is a rational number p i.e. $5+2\sqrt{7} = p$	1
	$\Rightarrow \sqrt{7} = \frac{p-5}{2}$	1/2
	Which is a contradiction as RHS is a rational but LHS is irrational.	
	Hence $5+2\sqrt{7}$ can not be rational, so irrational.	1/2
	OR	
	Check whether 12^n can end with the digit 0 for any natural number n.	
	Ans: Prime factors of 12 are 2 × 2 × 3	1
	Since 5 is not a factor, so 12^n can not end with 0.	1
26.	If A, B and C are interior angles of a ΔABC, then show that $\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$.	
	Ans: $A + B + C = 180°$, $\therefore \frac{B+C}{2} = 90° - \frac{A}{2}$	1
	$\therefore \cot\left(\frac{B+C}{2}\right) = \cot\left(90° - \frac{A}{2}\right) = \tan\frac{A}{2}$	1

SECTION – C

Question numbers 27 to 34 carry 3 marks each.

27. In Figure-8, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of the circle is $6\sqrt{2}$ cm, find the area of shaded region.

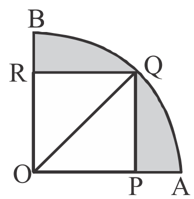


Figure 8

Ans: Let side of square be 'a' cm $\therefore$ $a^2 + a^2 = \left(6\sqrt{2}\right)^2$ $\Rightarrow$ a = 6 cm **1**

$\therefore$ Area of shaded region $= \pi r^2 \dfrac{90}{360} - a^2 = \dfrac{22}{7} \times \left(6\sqrt{2}\right)^2 \cdot \dfrac{1}{4} - 36$ **$1+\frac{1}{2}$**

$= \dfrac{396-252}{7} = \dfrac{144}{7}\text{cm}^2$ or 20.57 cm^2 **1/2**

28. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Ans: Constructing ΔABC with given dimensions **1**

Constructing the similar triangle. **2**

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Ans: Drawing a circle of radius 3.5 cm and centre O, and taking a point P such that OP = 7 cm **1**

Constructing two tangents. **2**

29. Prove that:

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta} = \cot\theta$$

Ans: L.H.S. $= \dfrac{\cos\theta\left(2\cos^2\theta - 1\right)}{\sin\theta\left(1 - 2\sin^2\theta\right)}$ **1/2**

$= \dfrac{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}{\sin\theta\left(1-2\sin^2\theta\right)}$ **1**

$= \dfrac{\cos\theta\left(1-2\sin^2\theta\right)}{\sin\theta\left(1-2\sin^2\theta\right)}$ **1**

= cos θ = R.H.S. **1/2**

30. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Ans: Let the fraction be $\frac{x}{y}$ — **1/2**

$\therefore \frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$ — **1/2+1/2**

$\Rightarrow 3x - y = 3 \text{ , } 4x - y = 8$ — **1/2**

Solving to get x = 5, y = 12 $\therefore$ Fraction is $\frac{5}{12}$ — **1**

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Ans: Let the present age of son be x years

$\therefore$ Father's present age = (3x + 3) years. — **1**

3 years hence, Son's age = (x + 3) years
and father's age = (3x + 6) years — **1/2**

$\therefore \quad 3x + 6 = 2(x + 3) + 10$ — **1**

$\Rightarrow \quad x = 10 \quad \therefore$ Son's age = 10 years,
Father's age = 33 years — **1/2**

31. Using Euclid's Algorithm, find the largest number which divides 870 and 258 leaving remainder 3 in each case.

Ans: HCF of (870 – 3) and (258 – 3) — **1**

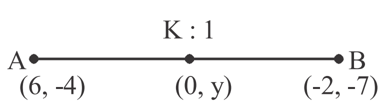
= 867 and 255

$867 = 255 \times 3 + 102$
$255 = 102 \times 2 + 51$
$102 = 51 \times 2 + 0$ — **$1\frac{1}{2}$**

$\therefore$ HCF = 51 — **1/2**

32. Find the ratio in which the y-axis divides the line segment joining the points (6, –4) and (–2, –7). Also find the point of intersection.

Ans:



Let the point P(0, y) on y-axis divides the line segment AB in K : 1 — **1**

$\therefore \quad 0 = \frac{-2K+6}{K+1} \Rightarrow K = 3 \quad \therefore$ Ratio is 3 : 1 — **1**

Also, $y = \frac{3(-7)+1(-4)}{3+1} = \frac{-25}{4}$ $\therefore$ Point of intersection is $\left(0, \frac{-25}{4}\right)$ — **1**

OR

Show that the points A(7, 10), B(–2, 5) and C(3, –4) are vertices of an isosceles right triangle.

Ans: Let the points be A(7, 10), B(–2, 5) and C(3, –4)

$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$ **1**

$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$ **1/2**

$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$ **1/2**

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle. **1**

33. In an A.P. given that the first term (a) = 54, the common difference (d) = –3, and the n^{th} term (a_n) = 0. Find n and sum of first n terms (S_n) of the A.P.

Ans: $a_n = 0$ **1/2**

$54 + (n-1)(-3) = 0$ **1**

$n = 19$

$S_{19} = \frac{19}{2}(54+0)$ **1**

$= 19 \times 27 = 513$ **1/2**

34. Read the following passage and answer the questions given at the end:

Diwali Fair

A game in a booth at Diwali fair involves using of spinner first. Then, if the spinner stops at an even number, the player is allowed to pick a marble from bag. The spinner and the marbles in the bag are represented in Figure-9

Prizes are given, when a black marble is picked. Shweta plays the game once.

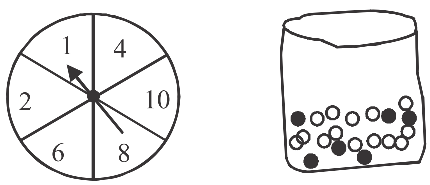


Figure 9

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Ans: (i) P(she will be allowed to pick a marble) = $\frac{5}{6}$ **$1\frac{1}{2}$**

(ii) P(getting a prize) = $\frac{6}{20}$ or $\frac{3}{10}$ **$1\frac{1}{2}$**

Both answers $\frac{6}{20}$ or $\frac{0}{20}$ for part (ii) in Q34 are to be treated correct as the bag contains marbles only.

SECTION – D

Question numbers 35 to 40 carry 4 marks each.

35. Sum of the areas of 2 squares is 544 m^2. If the difference of their perimeter is 32 m, find the sides of two squares.

Ans: Let ‘a’ and ‘b’ be the sides of two squares, with a > b.

then $a^2 + b^2 = 544$ and $4a - 4b = 32$ — $1+\frac{1}{2}$

or $a - b = 8 \;\therefore\; a = b + 8$ — 1

$\therefore\; (b+8)^2 + b^2 = 544 \;\Rightarrow\; 2b^2 + 16b - 480 = 0$ — 1

$\therefore\; b^2 + 8b - 240 = 0 \;\Rightarrow\; (b+20)(b-12) = 0 \;\Rightarrow\; b = 12$

$b = 12$ m $\Rightarrow a = 12 + 8 = 20$ m — 1/2

OR

A motorboat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans: Let speed of the stream be x km/h

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ — 2

$\Rightarrow 24(2x) = 324 - x^2$ or $x^2 + 48x - 324 = 0$ — 1

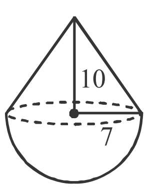
$\Rightarrow (x+54)(x-6) = 0 \Rightarrow x = 6$ — 1

$\therefore$ Speed of the stream = 6 km/h

36. A solid toy in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of its base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy.

(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

Ans: Volume of toy $= \frac{2}{3}\pi(7)^3 + \frac{1}{3}\pi(7)^2 \times 10\,cm^3$ — 1



$= \frac{1}{3} \times \frac{22}{7} \times 49\,(14 + 10) = 1232\,cm^3$ — 1

Area of Sheet = Surface area $= 2\pi(7)^2 + \pi(7)\sqrt{10^2 + 7^2}$ — 1

$= 308 + 22 \times 12.2 = 576.4 cm^2$ — 1

37. For the following data, draw a ‘less than’ ogive and hence find the median of the distribution.

Age (In years):	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

Ans: The points to be plotted for less than ogive are

(10, 5), (20, 20), (30, 40), (40, 65), (50 , 80), (60, 91), (70, 100) **2**

Drawing the ogive **$1+\frac{1}{2}$**

Getting median = 34 (approx) **1/2**

OR

The distribution given below shows that the number of wickets taken by bowler in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20-60	60-100	100-140	140-180	180-220	220-260
Number of bowlers :	7	5	16	12	2	3

Ans:

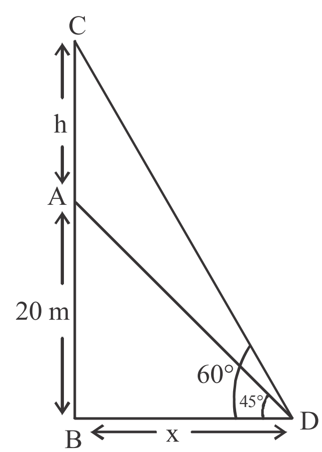
No. of wickets :	20-60	60-100	100-140	140-180	180-220	220-260	Sum	
(f_i) No. of bowlers :	7	5	16	12	2	3	45	
x_i	40	80	120	160	200	240		
u_i	−2	−1	0	1	2	3		**1/2**
$f_i x_i$	−14	−5	0	12	4	9	6	**1/2**
cf	7	12	28	40	42	45		**1/2**

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 120 + \frac{6 \times 40}{45} = 125.33$$ **$1+\frac{1}{2}$**

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h = 100 + \frac{22.5 - 12}{16} \times 40 = 126.25$$ **1**

38. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower $\left(\text{Use } \sqrt{3} = 1.73\right)$

Ans: Let h be the height of the tower **cor. fig. 1**



In right ΔABD

$$\left.\begin{aligned} \frac{20}{x} &= \tan 45° \\ x &= 20 \text{ m} \end{aligned}\right\}$$ **1**

In right ΔCBD

$$\frac{h+20}{20} = \tan 60°$$ **1**

$$h + 20 = 20\sqrt{3}$$

$$h = 20\left(\sqrt{3} - 1\right)$$ **1/2**

$$= 20 \times 0.73 = 14.60 \text{ m}$$ **1/2**

39. Prove that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two sides.

Ans: For correct Given, To Prove, Construction and figure — $\frac{1}{2} \times 4 = 2$

For correct proof — 2

40. Obtain other zeroes of the polynomial

$$P(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$$

If two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Ans: Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x), so $\left(x-\sqrt{5}\right)$ and $\left(x+\sqrt{5}\right)$ are factors of p(x). Thus $(x^2 - 5)$ is a factor of p(x). — 1

$\left(2x^4 - x^3 - 11x^2 + 5x + 5\right) \div \left(x^2 - 5\right) = 2x^2 - x - 1$ — $1\frac{1}{2}$

$2x^2 - x - 1 = (2x + 1)(x - 1)$ — 1

$\therefore$ Other zeroes of p(x) are $1, -\frac{1}{2}$ — 1/2

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?

Ans:

$$\begin{array}{r|l} & 2x+5 \\ x^2-4x+8 & 2x^3 - 3x^2 + 6x + 7 \\ - & 2x^3 \underset{+}{-} 8x^2 \underset{-}{+} 16x \\ \hline & 5x^2 - 10x + 7 \\ - & 5x^2 \underset{+}{-} 20x \underset{-}{+} 40 \\ \hline & 10x - 33 \end{array}$$

— 3

$\therefore$ We have to add $(33 - 10x)$ — 1

QUESTION PAPER CODE 30/5/3

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is	
	(a) 4 **(b)** ± 4 **(c)** -4 **(d)** 0	
	Ans: (b) ± 4	1
2.	Which of the following is *not* an A.P.?	
	(a) $-1.2, 0.8, 2.8, \ldots$ **(b)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$	
	(c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$ **(d)** $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$	
	Ans: (c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$	1
3.	The radius of a sphere (in cm) whose volume is 12π cm^3, is	
	(a) 3 **(b)** $3\sqrt{3}$ **(c)** $3^{2/3}$ **(d)** $3^{1/3}$	
	Ans: (c) $3^{2/3}$	1
4.	The distance between the points (m, –n) and (–m, n) is	
	(a) $\sqrt{m^2+n^2}$ **(b)** m + n	
	(c) $2\sqrt{m^2+n^2}$ **(d)** $\sqrt{2m^2+2n^2}$	
	Ans: (c) $2\sqrt{m^2+n^2}$	1
5.	In Figure 1, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is	
	(a) 3 cm	
	(b) 4 cm	
	(c) 2 cm	
	(d) $2\sqrt{2}$ cm	
	Ans: (b) 4 cm	1
6.	On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is	
	(a) $3x^2 + x - 12$ **(b)** $x^3 - 4x + 3$ **(c)** $x^2 + 3x - 4$ **(d)** $x^3 - 4x - 3$	
	Ans: (b) $x^3 - 4x + 3$	1

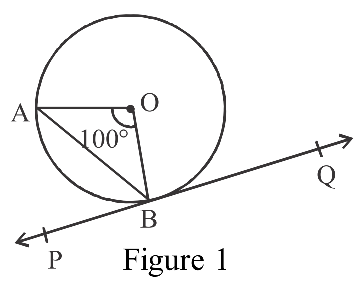


Figure 1

7. In Figure 2, DE || BC. If $\frac{AD}{DB} = \frac{3}{2}$ and AE = 2.7 cm, then EC is equal to

(a) 2.0 cm

(b) 1.8 cm

(c) 4.0 cm

(d) 2.7 cm

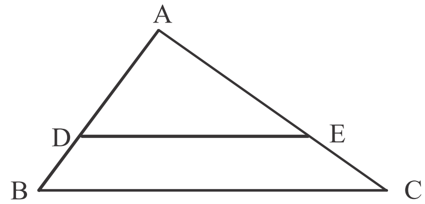


Figure 2

Ans: (b) 1.8 cm **1**

8. The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is

(a) (7, 0) **(b)** (5, 0) **(c)** (0, 0) **(d)** (3, 0)

Ans: (d) (3, 0) **1**

OR

The centre of a circle whose end points of a diameter are (– 6,3) and (6, 4) is

(a) (8, –1) **(b)** (4, 7) **(c)** $\left(0, \frac{7}{2}\right)$ **(d)** $\left(4, \frac{7}{2}\right)$

Ans: (c) $\left(0, \frac{7}{2}\right)$ **1**

9. The pair of linear equations

$\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is

(a) consistent **(b)** inconsistent

(c) consistent with one solution **(d)** consistent with many solutions

Ans: (b) inconsistent **1**

10. In Figure 3, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

(a) 50°

(b) 40°

(c) 60°

(d) 80°

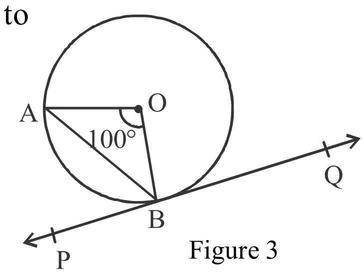


Figure 3

Ans: (a) 50° **1**

Fill in the blanks in question numbers 11 to 15.

11. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is __________.

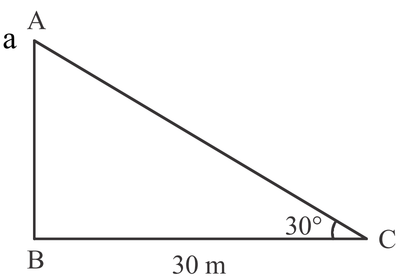
Ans: $\tan^2 A$ **1**

12. If the probability of an event E happening is 0.023, then $P(\overline{E}) =$ __________.

Ans: 0.977 **1**

13. All concentric circles are __________ to each other.

Ans: similar **1**

14.	The probability of an event that is sure to happen, is _________. **Ans:** 1	1
15.	AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is _________. **Ans:** 5 units	1
	Answer the following question numbers 16 to 20.	
16.	Write the value of $\sin^2 30° + \cos^2 60°$. **Ans:** $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$	1/2
	$= \frac{1}{2}$	1/2
17.	Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively. **Ans:** $x^2 + 3x + 2$	1
	OR	
	Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$? Justify your answer with reasons. **Ans:** No, degree of remainder < degree of divisor	1
18.	Find the sum of the first 100 natural numbers. **Ans:** $\frac{100}{2}[2+99] = 5050$	1/2+1/2
19.	The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other. **Ans:** $\frac{182 \times 13}{26} = 91$	1/2+1/2
20.	In Figure 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of tower.  **Ans:** $\frac{AB}{30} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m	1/2+1/2
	SECTION – B	
	Question numbers 21 to 26 carry 2 marks each.	
21.	A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. **Ans:** $\frac{\frac{1}{3}\pi r^2 (3h)}{\pi r^2 h}$	1
	$= \frac{1}{1} = 1 : 1$	1

22. In Figure 5, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

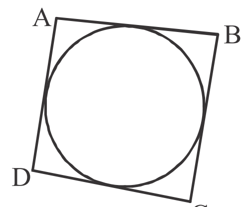
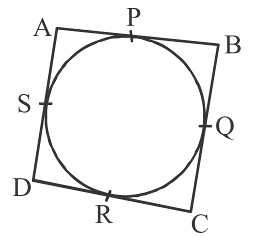


Figure 5

Ans: Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively. 1/2



$\therefore$ AP = AS, BP = BQ, DR = DS, CR = CQ 1

adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ) 1/2

$\therefore$ AB + CD = BC + AD

OR

In Figure 6, find the perimeter of ΔABC, if AP = 12 cm.

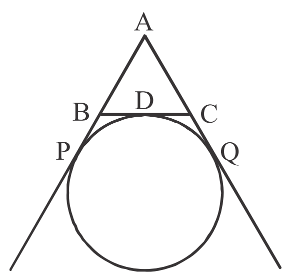


Figure 6

Ans: AP = AB + BP = AB + BD
AQ = AC + CQ = AC + CD 1

$\Rightarrow$ AP + AQ = AB + AC + (BD + CD) = AB + AC + BC

But AP = AQ $\therefore$ 2 AP = Perimeter of ABC 1/2

$\therefore$ Perimeter = 2(12) = 24 cm 1/2

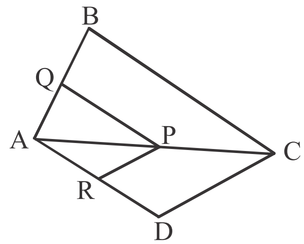
23. Find the mode of the following distribution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students:	4	6	7	12	5	6

Ans: Modal Group : 30 – 40 1/2

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 30 + \frac{5}{12} \times 10$$ 1

$= 34.17$ 1/2

24.	In the Figure 7, if PQ ∥ BC and PR ∥ CD, prove that $\frac{QB}{AQ} = \frac{DR}{AR}$.  Figure 7	
	Ans: $\frac{QB}{AQ} = \frac{PC}{AP}$ ...(i)	1
	$\frac{PC}{AP} = \frac{DR}{AR}$ ...(ii) From (i) and (ii)	1/2
	$\frac{QB}{AQ} = \frac{DR}{AR}$	1/2
25.	Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.	
	Ans: Let us assume that $5+2\sqrt{7}$ is not an irrational number.	
	$\therefore$ $5+2\sqrt{7}$ is a rational number p i.e. $5+2\sqrt{7} = p$	1
	$\Rightarrow \sqrt{7} = \frac{p-5}{2}$	1/2
	Which is a contradiction as RHS is a rational but LHS is irrational.	
	Hence $5+2\sqrt{7}$ can not be rational, so irrational.	1/2
	OR	
	Check whether 12^n can end with the digit 0 for any natural number n.	
	Ans: Prime factors of 12 are 2 × 2 × 3	1
	Since 5 is not a factor, so 12^n cannot end with 0.	1
26.	If A, B and C are interior angles of a ΔABC, then show that $\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$.	
	Ans: $A + B + C = 180°$, $\therefore \frac{B+C}{2} = 90° - \frac{A}{2}$	1
	$\therefore \cot\left(\frac{B+C}{2}\right) = \cot\left(90° - \frac{A}{2}\right) = \tan\frac{A}{2}$	1

SECTION – C

Question numbers 27 to 34 carry 3 marks each.

27. Prove that:

$$\left(\sin^4\theta - \cos^4\theta + 1\right)\operatorname{cosec}^2\theta = 2$$

Ans: L.H.S $= \left[\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^2\theta - \cos^2\theta\right) + 1\right]\operatorname{cosec}^2\theta$ — 1

$= \left[1\left(\sin^2\theta - \cos^2\theta\right) - 1\right]\operatorname{cosec}^2\theta$ — 1

$= 2\sin^2\theta \times \operatorname{cosec}^2\theta = 2$ — 1

28. Find the sum:

$(-5) + (-8) + (-11) + \ldots + (-230)$

Ans: $a = -5,\ d = -3,\ a_n = -230$ — 1/2

$\Rightarrow \quad -5 + (n-1) \times (-3) = 230$ — 1

$(n-1) = \dfrac{225}{3} = 75$

$n = 76$ — 1/2

$S_{76} = \dfrac{76}{2}\left[-5 + (-230)\right]$ — 1/2

$= 38(-235) = -8930$ — 1/2

29. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

Then construct a triangle whose sides are $\dfrac{3}{4}$ of the corresponding sides of ΔABC.

Ans: Constructing ΔABC with given dimensions — 1

Constructing the similar triangle. — 2

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Ans: Drawing a circle of radius 3.5 cm and centre O, and taking a point P such that OP = 7 cm — 1

Constructing two tangents. — 2

30. In Figure-8, ABCD is a parallelogram. A semicircle with centre O and the diameter AB has been drawn and it passes through D. If AB = 12 cm and OD ⊥ AB, then find the area of the shaded region. (Use π = 3.14)

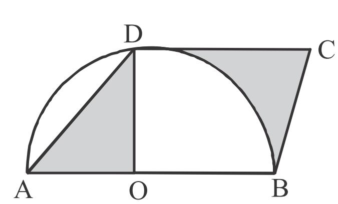
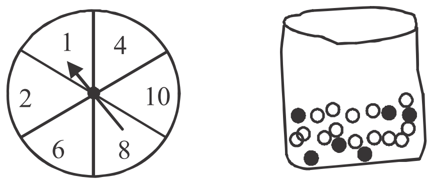


Figure 8

	Ans: Area of shaded portion = Ar of ||gm – Ar of Quadrant	1
	$= 12 \times 6 - \frac{1}{4} \times 3.14 \times 6 \times 6$	1
	$= 43.75 \text{ cm}^2$	1
31.	Read the following passage and answer the questions given at the end: **Diwali Fair** A game in a booth at Diwali fair involves using of spinner first. Then, if the spinner stops at an even number, the player is allowed to pick a marble from bag. The spinner and the marbles in the bag are represented in Figure-9 Prizes are given, when a black marble is picked. Shweta plays the game once.	
	 Figure 9	
	(i) What is the probability that she will be allowed to pick a marble from the bag? (ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?	
	Ans: (i) P(she will be allowed to pick a marble) = $\frac{5}{6}$	$1\frac{1}{2}$
	(ii) P(getting a prize) = $\frac{6}{20}$ or $\frac{3}{10}$	$1\frac{1}{2}$
	Both answers $\frac{6}{20}$ or $\frac{0}{20}$ for part (ii) in Q31 are to be treated correct as the bag contains marbles only.	
32.	A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.	
	Ans: Let the fraction be $\frac{x}{y}$	1/2
	$\therefore \frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$	1/2+1/2
	$\Rightarrow 3x - y = 3 , 4x - y = 8$	1/2
	Solving to get x = 5, y = 12 $\therefore$ Fraction is $\frac{5}{12}$	1

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Ans: Let the present age of son be x years

$\therefore$ Father's present age = (3x + 3) years. **1**

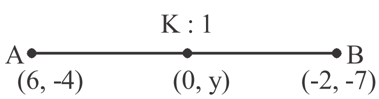
3 years hence, Son's age = (x + 3) years
and father's age = (3x + 6) years } **1/2**

$\therefore \quad 3x + 6 = 2(x + 3) + 10$ **1**

$\Rightarrow \quad x = 10 \quad \therefore$ Son's age = 10 years,
Father's age = 33 years **1/2**

33. Find the ratio in which the y-axis divides the line segment joining the points (6, –4) and (–2, –7). Also find the point of intersection.

Ans:



Let the point P(0, y) on y-axis divides the line segment AB in K : 1 **1**

$\therefore \quad 0 = \dfrac{-2K+6}{K+1} \Rightarrow K = 3 \quad \therefore$ Ratio is 3 : 1 **1**

Also, $y = \dfrac{3(-7)+1(-4)}{3+1} = \dfrac{-25}{4}$ $\therefore$ Point of intersection is $\left(0, \dfrac{-25}{4}\right)$ **1**

OR

Show that the points (7, 10), (–2, 5) and (3, –4) are vertices of an isosceles right triangle.

Ans: Let the points be (7, 10), (–2, 5) and (3, –4)

$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$ **1**

$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$ **1/2**

$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$ **1/2**

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle. **1**

34. Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.

Ans: Any positive integer 'n' can be of the form 3m, 3m + 1, 3m + 2 **1**

$\therefore \quad n^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3q,$

or $\quad n^2 = (3m+1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1 = 3q + 1,$

or $\quad n^2 = (3m+2)^2 = 9m^2 + 12m + 3 + 1$
$= 3(3m^2 + 4m + 1) + 1 = 3q + 1$ } $1\frac{1}{2}$

Hence square of any positive integer is either of the form 3q or 3q + 1 for some integer q. **1/2**

SECTION – D

Question numbers 35 to 40 carry 4 marks each.

35. Sum of the areas of 2 squares is 544 m^2. If the difference of their perimeter is 32 m, find the sides of two squares.

Ans: Let 'a' and 'b' be the sides of two squares, with a > b.

then $a^2 + b^2 = 544$ and $4a - 4b = 32$ $1\frac{1}{2}$

or $a - b = 8 \;\therefore\; a = b + 8$ 1

$\therefore\; (b + 8)^2 + b^2 = 544 \;\Rightarrow\; 2b^2 + 16b - 480 = 0$ 1

$\therefore\; b^2 + 8b - 240 = 0 \;\Rightarrow\; (b + 20)(b - 12) = 0 \;\Rightarrow\; b = 12$

$b = 12$ m $\Rightarrow\; a = 12 + 8 = 20$ m 1/2

OR

A motorboat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans: Let speed of the stream be x km/h

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ 2

$\Rightarrow\; 24(2x) = 324 - x^2$ or $x^2 + 48x - 324 = 0$ 1

$\Rightarrow\; (x + 54)(x - 6) = 0 \;\Rightarrow\; x = 6$ 1

$\therefore$ Speed of the stream = 6 km/h

36. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (In years):	0-10	10-20	20-30	0-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

Ans: The points to be plotted for less than ogive are

(10, 5), (20, 20), (30, 40), (40, 65), (50 , 80), (60, 91), (70, 100) 2

Drawing the ogive $1\frac{1}{2}$

Getting median = 34 (approx) 1/2

OR

The distribution given below shows that the number of wickets taken by bowler in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20-60	60-100	100-140	140-180	180-220	220-260
Number of bowlers :	7	5	16	12	2	3

No. of wickets :	20-60	60-100	100-140	140-180	180-220	220-260	Sum	
(f_i) No. of bowlers :	7	5	16	12	2	3	45	
x_i	40	80	120	160	200	240		
u_i	−2	−1	0	1	2	3		1/2
f_ix_i	−14	−5	0	12	4	9	6	1/2
cf	7	12	28	40	42	45		1/2

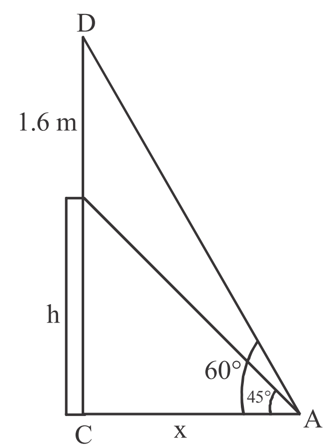
Mean = $a + \frac{\sum f_iu_i}{\sum f_i} \times h = 120 + \frac{6 \times 40}{45} = 125.33$ **$1\frac{1}{2}$**

Median = $l + \frac{\frac{N}{2} - c}{f} \times h = 100 + \frac{22.5 - 12}{16} \times 40 = 126.25$ **1**

37. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statute is 60° and from the same point the angle of elevation of the top of pedestal is 45°. Find the height of the pedestal. (Use $\sqrt{3}$ = 1.73)

Ans: For correct figure. **1**

Let h m be the height of pedestal



Then from figure, $\frac{h}{x} = \tan 45° = 1$

and $\frac{h+1.6}{x} = \tan 60° = \sqrt{3}$ **1 + 1**

$\Rightarrow \left(\sqrt{3} - 1\right)h = 1.6$ **1/2**

$\Rightarrow h = \frac{160}{73} = 2.19$ m (approx) **1/2**

38. Obtain other zeroes of the polynomial

$$P(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$$

If two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Ans: Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x), so $\left(x - \sqrt{5}\right)$ and $\left(x + \sqrt{5}\right)$ are factors of p(x). Thus $(x^2 - 5)$ is a factor of p(x). **1**

$\left(2x^4 - x^3 - 11x^2 + 5x + 5\right) \div \left(x^2 - 5\right) = 2x^2 - x - 1$ **$1\frac{1}{2}$**

$2x^2 - x - 1 = (2x + 1)(x - 1)$ **1**

$\therefore$ Other zeroes of p(x) are $1, -\frac{1}{2}$ **1/2**

	OR	
	What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?	
	Ans:	
	$$\begin{array}{r} 2x + 5 \\ x^2 - 4x + 8 \overline{)\,2x^3 - 3x^2 + 6x + 7\,(} \\ - \underset{-}{2x^3} \underset{+}{-8x^2} \underset{-}{+16x} \\ \hline 5x^2 - 10x + 7 \\ - \underset{-}{5x^2} \underset{+}{-20x} \underset{-}{+40} \\ \hline 10x - 33 \end{array}$$	**3**
	$\therefore$ We have to add $(33 - 10x)$	**1**
39.	In a cylindrical vessel of radius 10 cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm, then find the rise in the level of water in the vessel.	
	Ans: Volume of raised water in cylinder = Volume of 9000 spherical balls	**1**
	$\pi(100)^2 H = 9000 \times \frac{4}{3} \times \pi \times (0.5)^3$	**2**
	$\therefore$ H = 15 cm	**1**
40.	If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.	
	Ans: For correct Given, To Prove, Constructions and figure	$\frac{1}{2} \times 4 = 2$
	For correct proof	**2**